Alfvén pulse at chromospheric footpoints of magnetic loops and generation of the super-Dreicer electric field

N. A. Emelyanov[1,2], Vl. V. Kocharovsky[1]

[1] *A.V. Gaponov-Grekhov Institute of Applied Physics of the Russian Academy of Sciences, Nizhny Novgorod, Russia*

[2] *Pulkovo Astronomical Observatory, Russian Academy of Sciences, Pulkovo, Russia*

n.emelyanov@ipfran.ru

Abstract

A self-similar solution of the linearised magnetohydrodynamic equations describing the propagation of the Alfvén pulse in an axially symmetric magnetic tube of variable diameter is obtained. Using perturbation theory and the solution found for the case of a homogeneous in a cross section magnetic tube, the electric field component induced by the non-linear Alfvén wave and directed along the tube surface, i.e. accelerating particles along the magnetic field, is determined. For the chromospheric footpoints of the magnetic loops, whose configuration is given by the barometric law of plasma pressure decay, the conditions for achieving the super-Dreicer electric field limit necessary to drive the accelerated high-energy electrons into the coronal part of the loop are established.

1 Introduction

Acceleration of charged particles in the chromospheric and coronal plasma is one of the main channels for free magnetic energy release during solar flares [1, 2, 3]. Measurements of radiation intensity in the ultraviolet and X-ray bands during the most powerful flares indicate that a large number of particles are involved in the acceleration process, comparable to or even exceeding the total number of particles contained in a volume on the order of the volume of the coronal part of the magnetic loop [4]. This is difficult to explain within the framework of the standard models, where the acceleration process occurs in a small region of the coronal part of the flare loop.

The injection of particles from the chromospheric footpoints into the upper parts of the loops potentially solves the above problem and explains the available data. The process described is confirmed in particular by observations showing that particle acceleration and plasma heating can take place directly in the chromospheric bases of magnetic loops, where the plasma density is much higher (see e.g. [5, 6]). However, this requires the presence of a strong electric field that exceeds the so-called Dreicer limit [7, 8].

The papers [9, 10] proposed a scenario in which a strong electric field can be generated by a non-linear Alfvén pulse resulting from the development of the Rayleigh-Taylor instability in the chromospheric base of the magnetic loop. However, these works did not take into account the change in the cross section of the magnetic tube along which the Alfvén wave propagates.

In the present work we consider a more general formulation of the problem, not limited by the cylindrical shape of the magnetic tube and some other approximations made in [9], and show that a change in the geometry of the magnetic loop is essential for the dynamics of the Alfvén pulse and the generation of the super-Dreicer electric field. To this end, the linear and non-linear modes of wave propagation are studied analytically, and the conditions for the transition to super-Dreicer acceleration of the electrons are found in the model of barometric plasma pressure decay.

The paper is structured as follows: in Section 2, the propagation of the Alfvén pulse in an axially symmetric magnetic tube with arbitrary plasma parameters is considered in the framework of single-fluid ideal magnetic hydrodynamics (MHD) in a linear approximation, and a self-similar solution is found for the case of a longitudinal component of the magnetic field homogeneous along the tube cross section; Section 3 is devoted to the investigation of the conditions of applicability of the obtained linear solution to the description of the propagation of the Alfvén wave in the chromospheric footpoints. Section 4 estimates the non-linear electric field component responsible for the acceleration of particles along the magnetic field lines of the initial coronal loop, based on the found self-similar solution for the Alfvén pulse; Section 5 is the conclusion.

2 Alfvén pulse propagation in an axially symmetric magnetic tube

In [9], the propagation of an Alfvén wave in a circular cylindrical magnetic tube of constant diameter was studied and reduced to the description of a one-dimensional string oscillations. As will be shown below, a similar reduction to a simple problem is also possible in the case of a more general geometry with a variable magnetic tube cross-section and arbitrary plasma parameters.

The configuration under consideration occurs naturally near regions of enhanced magnetic field strength (magnetic plugs), where the vertical component of the field varies with height. It may be an element of larger-scale magnetic structures in the solar atmosphere (the footpoints of magnetic loops, spicules, coronal holes, etc.) [11, 12] and an independent formation as well [13]. Analytical calculations and observations show that magnetic tubes are the channel for the transport of MHD waves, including Alfvén waves (see e.g. [14, 15, 16, 17]), from the upper layers of the photosphere to the transition region, where MHD waves, damped in the non-linear regime, can transfer their energy to the plasma and thus act as one of the possible sources of solar corona heating [11, 12, 13]. Therefore, a detailed analysis of MHD oscillations in the variable cross-section region of chromospheric magnetic tubes seems necessary.

We consider the propagation of an Alfvén wave in an axially symmetric tube and show that, under sufficiently weak constraints, we can obtain a self-similar solution in the form of running pulses whose shape change is determined by geometrical factors of the initial configuration of the magnetic tube.

Let us write down the general equations of ideal single-fluid magnetic hydrodynamics [18]:

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}), \tag{1a}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v} = -\frac{1}{\rho}\nabla P - \frac{1}{4\pi\rho}(\vec{B} \times (\nabla \times \vec{B})), \tag{1b}$$

$$\frac{\partial \rho}{\partial t} + \nabla * \rho\vec{v} = 0, \tag{1c}$$

$$\frac{dS}{dt} = 0. \tag{1d}$$

Here $\vec{B}$ is magnetic field induction, $\vec{v}$ is plasma flow velocity, ρ is plasma density, P is plasma pressure, S is plasma entropy, operator $\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{v}\nabla)$.

We will assume the plasma to be incompressible, i.e., the condition $\nabla * \vec{v} = 0$ is fulfilled. For an axially symmetric magnetic tube, all these quantities have no dependence on the azimuthal angle ϕ and, consequently, all derivatives are equal to zero: $\frac{\partial}{\partial \phi} = 0$. In the linear azimuthal component B_ϕ approximation, we will assume that there is no radial plasma flow: $v_r \equiv 0$ (the validity of this approximation is discussed in Section 3). Then, from the incompressibility condition $\nabla * \vec{v} = 0$, it follows that the longitudinal component of the plasma flow velocity is also absent, $v_z \equiv 0$, and from the equation for magnetic induction (1a) we have $\frac{\partial B_z}{\partial t} = 0$, $\frac{\partial B_r}{\partial t} = 0$, i.e., the longitudinal and radial components of the magnetic field $B_z(r,z)$ and $B_r(r,z)$ are completely determined by the initial values, as well as the plasma density $\rho(r,z)$.

This leaves only two varying azimuthal components, v_ϕ and B_ϕ, which determine the dynamics of the Alfvén wave. Let us project the corresponding MHD equations onto the ϕ axis (see A.1):

$$\begin{cases} \frac{\partial B_\phi}{\partial t} = B_z\frac{\partial v_\phi}{\partial z} + B_r\frac{\partial v_\phi}{\partial r} - v_\phi\frac{B_r}{r}, \\ \frac{\partial v_\phi}{\partial t} = \frac{1}{4\pi\rho}(B_z\frac{\partial B_\phi}{\partial z} + B_r\frac{\partial B_\phi}{\partial r} + B_\phi\frac{B_r}{r}). \end{cases} \tag{2}$$

Let us differentiate the first equation by time and substitute the second equation into it. The result is:

$$\frac{\partial^2 B_\phi}{\partial t^2} = \frac{B_z^2}{4\pi\rho}\frac{\partial^2 B_\phi}{\partial z^2} + 2\frac{B_z B_r}{4\pi\rho}\frac{\partial^2 B_\phi}{\partial z \partial r} + \frac{B_r^2}{4\pi\rho}\frac{\partial^2 B_\phi}{\partial r^2}$$
$$+\frac{B_z}{4\pi}\frac{\partial}{\partial z}(\frac{B_z}{\rho})\frac{\partial B_\phi}{\partial z} + \frac{B_r}{4\pi}\frac{\partial}{\partial r}(\frac{B_z}{\rho})\frac{\partial B_\phi}{\partial z} + \frac{B_z}{4\pi}\frac{\partial}{\partial z}(\frac{B_r}{\rho})\frac{\partial B_\phi}{\partial r} + \frac{B_r}{4\pi}\frac{\partial}{\partial r}(\frac{B_r}{\rho})\frac{\partial B_\phi}{\partial r}$$
$$+\frac{B_z}{4\pi}\frac{\partial}{\partial z}(\frac{B_r}{r\rho})B_\phi + \frac{B_r}{4\pi}\frac{\partial}{\partial r}(\frac{B_r}{r\rho})B_\phi - \frac{B_r^2}{4\pi\rho}\frac{B_\phi}{r^2}. \tag{3}$$

This equation describes the propagation of an Alfvén wave in a magnetic tube with arbitrary dependencies $B_r(r,z)$, $B_z(r,z)$ and $\rho(r,z)$. However, in its present form it is difficult to analyse. Let us choose new variables $\xi(r,z)$ and $\eta(r,z)$ so that the wave equation has the simplest form. To do this, we use the method of characteristics [19] and make a characteristic equation, which is written as follows:

$$B_z^2 dr^2 - 2B_z B_r dz dr + B_r^2 dz^2 = 0. \tag{4}$$

Its solution, obviously, is the equation of the line of the magnetic surface tube:

$$\frac{dr}{B_r(r,z)} = \frac{dz}{B_z(r,z)}. \tag{5}$$

Let us use the expression for the magnetic induction $\vec{B}$ by the vector potential $\vec{A}$:

$$\vec{B} = \nabla \times \vec{A}. \tag{6}$$

Then the characteristic equation is rewritten in the form:

$$\frac{1}{r}\frac{\partial(rA_\phi)}{\partial r}dr + \frac{\partial A_\phi}{\partial z}dz = 0. \tag{7}$$

As can be easily seen, $d(rA_\phi) = 0$ and, therefore, $rA_\phi = const$, i.e., rA_ϕ is the integral of the characteristic equation. Thus there is a manifold of solutions which describe the set of nested magnetic tubes defined by its constant value rA_ϕ. Setting the value of the integral distinguishes from this manifold of solutions a particular dependence of its radius on the longitudinal coordinate: $r|_{\xi=const} = r(z)$.

Let us choose as new coordinates the values expressed by the old ones:

$$\xi = rA_\phi(r,z), \tag{8a}$$
$$\eta = z. \tag{8b}$$

After replacing the variables we have $B_\phi(t,r,z) = B_\phi(t,\xi(r,z),\eta(r,z)) \equiv u(t,\xi,\eta)$. Note that the obtained integral (8a) of the characteristic equation has a simple physical meaning and expresses the conservation law of the magnetic flux $\Phi = \oint_\gamma \vec{A}d\vec{l} = \int_0^{2\pi} A_\phi(r,z)rd\phi = 2\pi rA_\phi = 2\pi\xi$. In addition, this quantity can be related to the Z-component of the angular momentum of the magnetic field $M_z = r(\frac{e}{c}A_\phi)$, the conservation of which is a consequence of the axial symmetry of the system.

In the new coordinates, the equation for the Alfvén wave takes the form (see A.2):

$$\frac{\partial^2 u}{\partial t^2} = \frac{B_z^2}{4\pi\rho}\frac{\partial^2 u}{\partial \eta^2} + \frac{B_z}{4\pi}\frac{\partial}{\partial \eta}(\frac{B_z}{\rho})\frac{\partial u}{\partial \eta} + [\frac{B_z}{4\pi}\frac{\partial}{\partial \eta}(\frac{B_r}{r\rho}) - \frac{1}{r^2}\frac{B_r^2}{4\pi\rho}]u. \tag{9}$$

We will look for the solution of this wave equation in the following form:

$$u = f(t,\xi,\eta)\exp(\frac{1}{2}\int^\eta \frac{1}{B_z}\frac{\partial B_z}{\partial \eta}d\eta). \tag{10}$$

Then, carrying out simple calculations (see A.3), we obtain:

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial \eta}(c_A^2\frac{\partial f}{\partial \eta}) + \frac{f}{\tau^2}, \tag{11}$$

where we introduce notations for the square of the Alfvén velocity $c_A^2 = \dfrac{B_z^2}{4\pi\rho}$ and the parameter τ, having the dimension of time and given by the expression:

$$\frac{1}{\tau^2} = \frac{\partial}{\partial\eta}(\frac{c_A^2}{L}) - \frac{c_A^2}{L^2}. \tag{12}$$

The value L written in the expression for τ has the dimension of length and determines the characteristic scale of the longitudinal change of the function $\varkappa$:

$$\frac{1}{L(\xi,\eta)} = \frac{1}{2\varkappa(\xi,\eta)}\frac{\partial\varkappa(\xi,\eta)}{\partial\eta}. \tag{13}$$

The introduced function $\varkappa(\xi,\eta) = \dfrac{(\vec{B} * \vec{S})}{\Phi}$ characterises the ratio of the edge value of the longitudinal component of the magnetic field B_z for each magnetic tube with a fixed value of magnetic flux Φ to its cross-sectional mean value. In other words, this function shows how close the selected magnetic tube is to the periphery of the system. Here $\vec{S} = \pi r^2 \vec{z}_0$ is the vector cross-sectional area of the magnetic tube corresponding to a fixed value of magnetic flux Φ. The unit vector $\vec{z}_0$ is directed along the OZ axis.

Thus, in the general case, assuming only axial symmetry of the magnetic tube with plasma, the derivatives on the variable ξ, which now enters the equation (11) only as a parameter, are completely excluded by appropriate substitution of the variables. In other words, the linear part of the Alfvén wave propagation problem has been reduced to the oscillation of a one-dimensional string whose properties are determined by a given initial distribution of the magnetic field components $B_r(r,z)$, $B_z(r,z)$ and the plasma density $\rho(r,z)$.

If the longitudinal component of the magnetic field B_z does not depend on the radius r, but is an arbitrary function of the longitudinal coordinate z, then the condition $\nabla * \vec{B} = 0$ implies that the radial component $B_r(r,z) = -\dfrac{r}{2}\dfrac{\partial B_z(z)}{\partial z}$.

On the other hand, for the found integral (8a) $\xi = rA_\phi(r,z)$:

$$\frac{\partial\xi}{\partial z} = r\frac{\partial A_\phi}{\partial z} = -rB_r = \frac{r^2}{2}\frac{\partial B_z}{\partial z}. \tag{14}$$

Choosing the natural for this case calibration of the vector potential $A_\phi(r,z) = \dfrac{1}{2}rB_z(z)$, we get:

$$\xi(r,z) = \frac{r^2}{2}B_z(z). \tag{15}$$

Then, by definition, $\varkappa \equiv 1$ and, as it can be easily seen, $\dfrac{1}{\tau^2} \equiv 0$. As a result, the wave equation (11) takes a simple form:

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial}{\partial\eta}(c_A^2\frac{\partial f}{\partial\eta}). \tag{16}$$

There are many constructive methods for solving this type of equation. In the simplest case of a weak dependence of the Alfvén velocity $c_A^2(\eta)$ on the longitudinal coordinate, we can assume that

$$\frac{\partial^2 f}{\partial t^2} = c_A^2\frac{\partial^2 f}{\partial\eta^2}. \tag{17}$$

Finally, using the well-known d'Alambert method, we obtain that for a given initial perturbation of the field $\psi(\xi,\eta)$, the solution of the equation (17) has the following form:

$$f = \frac{\psi(\xi,\eta - c_A(\eta)t) + \psi(\xi,\eta + c_A(\eta)t)}{2}. \tag{18}$$

Thus, by relating the initial value of the azimuthal component of the magnetic field $B_{\phi 0}(r,z)$, determined by the current distribution in the magnetic tube, to the initial perturbation $\psi(\xi,\eta)$ in the string oscillation problem (see ref. A.4), one can obtain a self-similar solution for the Alfvén perturbation at all subsequent times as the sum of two 'running' pulses (+ и -):

$$B_\phi(t,r,z) = \sum_{\pm}\frac{1}{2}B_{\phi 0}(r\sqrt{\frac{B_z(z)}{B_z(z \pm c_A(z)t)}}, z \pm c_A(z)t)\exp{(\frac{1}{2}\int_{z\pm c_A(z)t}^{z}\frac{1}{B_z}\frac{\partial B_z}{\partial z'}dz')}. \tag{19}$$

3 Non-linear properties of the Alfvén pulse and a barometric model of the chromospheric part of the magnetic loop

Let us estimate the non-linear corrections due to the presence of the plasma flow velocity components v_r and v_z. Write the Euler equation and project it on the OZ axis:

$$\frac{\partial v_z}{\partial t} \approx -\frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{1}{4\pi\rho}(B_r(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}) + B_\phi \frac{\partial B_\phi}{\partial z}). \quad (20)$$

Assuming that the appearance of the longitudinal component of the velocity v_z is determined only by the action of the Alfvén pulse, we approximate:

$$\frac{\partial v_z}{\partial t} \approx -\frac{B_\phi}{4\pi\rho}\frac{\partial B_\phi}{\partial z}. \quad (21)$$

As a result, using the self-similar solution found (19), we can make the following estimation:

$$|v_z| \approx \frac{1}{c_A}\frac{B_\phi^2}{4\pi\rho} = (\frac{B_\phi}{B_z})^2 c_A. \quad (22)$$

In the case, when $(\frac{B_\phi}{B_z})^2 \ll 1$, we get $|v_z| \ll c_A$. From the incompressibility condition $\nabla * \vec{v} = 0$ we also have: $v_r \sim \frac{l_r}{l_z} v_z$, where l_r, l_z are characteristic scales of the arising flows. Thus, in the above case, at a not too large radial scale l_r, the plasma motions do not lead to significant changes in the configuration of the magnetic tube during the passage time of the Alfvén pulse and, consequently, to distortions of the pulse itself, since their velocities are small compared to the Alfvén velocity v_z, v_r, confirming the validity of the linear approximation.

However, the very appearance of non-linear components of the plasma velocity as a result of the passage of the Alfvén wave also leads to the appearance of non-linear components of the electric field components, which can inject plasma from the chromospheric footpoints into the coronal regions of the loop and accelerate particles. This issue will be discussed in more detail in the next section.

Note that for values of the Alfvén velocity $c_A \sim 10^3$ km s^{-1} with a not too small ratio $(\frac{B_\phi}{B_z})^2 \sim 10^{-1} - 10^{-2}$, which can be achieved in magnetic loops with currents $\gtrsim 10^{10} A$ and characteristic diameter of magnetic tubes $10^2 - 10^3$ km [6], the obtained estimated value of the non-linear vertical component of the plasma velocity v_z is of the order of ~ 10 km s^{-1}. This value is confirmed by the observation of chromospheric plasma injection into the coronal parts of loops (see e.g. [5, 17]).

Consider the propagation of the Alfvén pulse in an axially symmetric magnetic tube of the chromospheric footpoint of a magnetic loop, where plasma concentration and pressure can be sufficiently large.

From the equilibrium condition one should expect that the configuration of the chromospheric footpoint is determined by the balance of the magnetic pressure $\frac{B^2}{8\pi}$ inside the tube (the pressure of the plasma itself inside the tube is considered negligibly small) and the external gas-kinetic pressure P_e.

If the longitudinal component of the magnetic field B_z is dominant, this condition has the form: $\frac{B_z^2}{8\pi} \approx P_e$. Suppose further that this component $B_z(z)$ depends only on the longitudinal (vertical) coordinate, and that the external pressure in equilibrium is given by the quasi-barometric formula:

$$P_e(z) = P_0 \exp(-\int_0^z \frac{dz'}{H(z')}), \quad (23)$$

where the reduced height $H(z) = \frac{k_B T(z)}{m_i g}$ is introduced, k_B is the Boltzmann constant, $T(z)$ is the plasma temperature, m_i is the mass of ions, g is the acceleration of free fall at the surface of the Sun[1]. Then from the equilibrium condition we have:

$$B_z(z) = B_0 \exp(-\int_0^z \frac{dz'}{2H(z')}). \quad (24)$$

[1]The value of $H(z)$ actually varies weakly within the chromosphere [20], remaining on the order of a few hundred kilometres.

In [9] it was shown that for such a field configuration, by virtue of the conservation law of magnetic flux, there is an exponential expansion of the magnetic tube ($a(z) = a_0 \exp \int_0^z \frac{dz'}{4H(z')}$, where $a(z)$ is the tube radius), which can become unstable with respect to the growth of the Rayleigh-Taylor mode providing the generation of the Alfvén pulse.

For the sake of clarity, we assume that the plasma density at the chromospheric footpoint of the loop decreases according to the same exponential law as the external pressure. In this case, the changes in density and magnetic field are consistent, so that the Alfvén velocity $c_A^2 = \dfrac{B_z^2}{4\pi\rho}$ varies weakly with height with a slow dependence on the longitudinal coordinate of the plasma temperature. Such a consideration is quite acceptable at altitudes below the transition region between the chromosphere and the solar corona.

Under these assumptions, the approximation used in the previous section is applicable to describe the propagation of the Alfvén pulse. Substituting the specific dependence of the longitudinal component of the magnetic field (24) into the expression for the Alfvén perturbation (19) obtained there, and leaving only the upward pulse, we have:

$$B_\phi(t, r, z) = \frac{1}{2} e^{-\int_{z-c_A t}^{z} \frac{dz'}{4H(z')}} B_{\phi 0}(r e^{-\int_{z-c_A t}^{z} \frac{dz'}{4H(z')}}, z - c_A(z)t). \tag{25}$$

Thus, the Alfvén pulse weakly changes in the layer of thickness of the order of H adjacent to the photosphere as it moves upward toward the corona, but stretches significantly in the transverse direction, following the exponential expansion of the magnetic tube, and decays exponentially as it travels a distance greater than H (see Fig.1).

According to the expression (19), there is also a second, exponentially increasing, pulse moving downwards to the base of the tube. However, the ratios $\dfrac{B_\phi}{B_\phi}$ and $\dfrac{B_\phi^2}{8\pi P}$ for such a pulse exponentially decrease when the distance travelled is greater than H, which weakens the generation of the non-linear electric field component discussed below in Section 4.

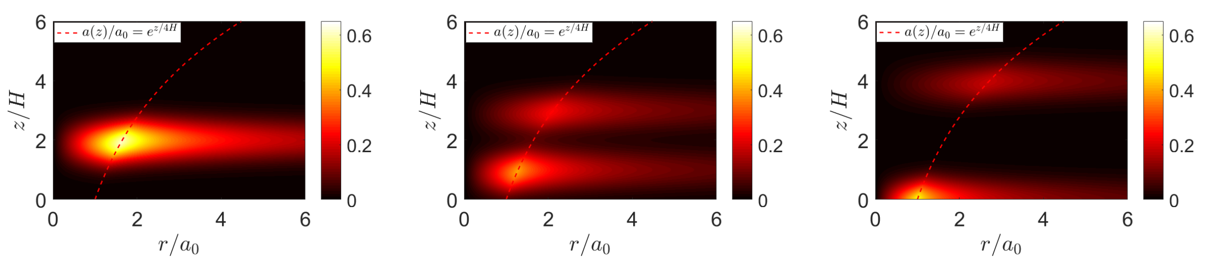

Figure 1: Self-similar solution for the Alfvén pulse (19) in the case of an exponentially expanding magnetic tube ($H = const$) with a homogeneous over the cross-section electric current distribution: $j(r) = const$ at $r < a(z)$ and $j(r) = 0$ at $r \geqslant a(z)$, and a Gaussian distribution ($I(z) \sim \exp(-\dfrac{(z - z_0)^2}{\sigma^2})$, in this figure $z_0 = 2$, $\sigma^2 = 0.5$) along the longitudinal coordinate z. The distribution of the dimensionless magnetic field component B_ϕ at different moments of time $\tau = 0, 1, 2$, where $\tau = \dfrac{c_A t}{H}$, is shown. The dashed curve is the radius of the magnetic tube.

4 Generation of the accelerating electric field by a non-linear Alfvén wave

To solve the problem of the acceleration of plasma particles in the chromospheric footpoints of magnetic loops, the most important thing is to estimate the magnitude of the electric field induced by the Alfvén pulse.

Using the ideal conductivity condition, we express the electric field strength in terms of the local plasma velocity and the magnetic induction vector:

$$\vec{E} = -\frac{1}{c} \vec{v} \times \vec{B}. \tag{26}$$

Obviously, the particle-accelerating component of the electric field, directed along the initial configuration of the magnetic tube, is related to the plasma velocity component, directed normal to the lateral surface of the tube, and the azimuthal component of the magnetic field in the following way (see Fig.2):

$$E_{\|} = -\frac{1}{c} v_n B_\phi. \tag{27}$$

From this we can see that in the absence of the velocity components v_r and v_z, which is the case in the linear approximation made in Section 2 when analysing the propagation of the Alfvén pulse, the longitudinal component of the electric field is also absent and hence no plasma flow or particle acceleration occurs.

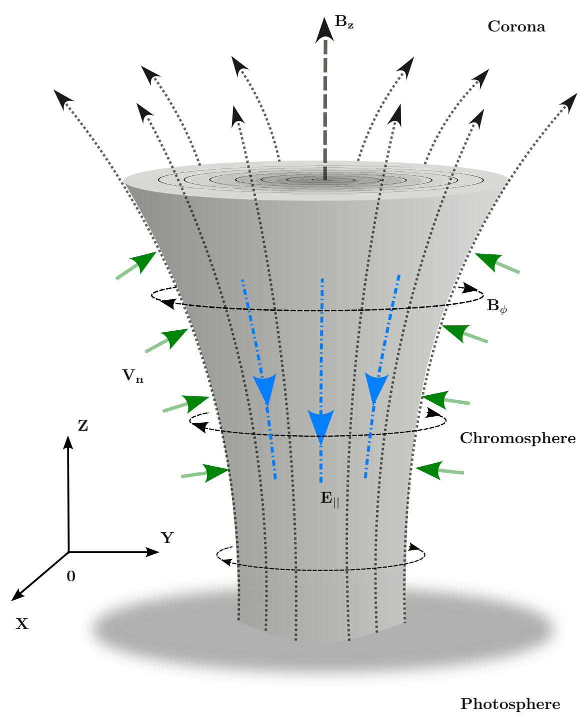

Figure 2: The expanding magnetic tube of the chromospheric footpoint of the coronal loop. The black arrows show the magnetic field lines, the green arrows show the normal component of the plasma flow velocity V_n, and the blue arrows show the non-linear electric field component $E_{||}$ directed along the tube surface.

However, as will be shown below, if the amplitude of the Alfvén wave is sufficiently large, a normal component of the plasma velocity arises, leading to the appearance of a non-linear B_ϕ longitudinal component of the electric field.

Let us write the Euler equation that determines the change in plasma velocity:

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho}\nabla P - \frac{1}{4\pi\rho}(\vec{B} \times (\nabla \times \vec{B})). \tag{28}$$

Projecting this equation onto the normal to the surface of the force tube, we obtain:

$$\frac{\partial v_n}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial n} - \frac{1}{\rho}\frac{\partial}{\partial n}(\frac{B^2}{8\pi}) + \frac{B^2}{4\pi\rho}\frac{1}{R}. \tag{29}$$

Here $\dfrac{\partial}{\partial n} = (\vec{n} * \nabla)$, where R is the radius of the magnetic field lines curvature. The vector $\vec{n}$ is the normal vector to the magnetic tube surface and is defined in the cylindrical coordinate system by the components $\vec{n} = (-\frac{B_r}{\sqrt{B_r^2+B_z^2}}, 0, \frac{B_z}{\sqrt{B_r^2+B_z^2}})$. In the stationary case or at small amplitude of the Alfvén wave we have:

$$-\frac{1}{\rho}\frac{\partial P}{\partial n} - \frac{1}{\rho}\frac{\partial}{\partial n}(\frac{B^2}{8\pi}) + \frac{B^2}{4\pi\rho}\frac{1}{R} = 0. \tag{30}$$

To obtain an estimation of the electric field magnitude, we will assume that the magnitude of the azimuthal component of the magnetic field B_ϕ is sufficiently large so that a change in the magnetic pressure cannot be compensated by a change in the gas pressure ($P \ll \frac{B_\phi^2}{8\pi}$). Suppose also that the characteristic scale of the Alfvén pulse is much smaller than the characteristic curvature of the magnetic field lines. Then, we approximate the equation for the normal component of the plasma velocity:

$$\frac{\partial v_n}{\partial t} \approx -\frac{1}{\rho}\frac{\partial}{\partial n}(\frac{B_\phi^2}{8\pi}). \tag{31}$$

Let us use the solution for the Alfvén wave in the case of an exponentially expanding tube (25) found in the previous section. Note that for the chosen magnetic tube ($\Phi = const$) the axial component of the magnetic field B_ϕ depends only on time as $B_\phi(\eta - c_A t)$ (for a short pulse with a characteristic length $\lambda \lesssim 4H$ the exponential multiplier can be ignored). Then we look for the solution in the same form and write, replacing the differentiation on the variable t by the differentiation on η:

$$-c_A\frac{\partial v_n}{\partial \eta} \approx -\frac{1}{\rho}\frac{\partial}{\partial n}(\frac{B_\phi^2}{8\pi}). \tag{32}$$

Let us integrate the obtained equation, taking into account that the normal component of the plasma flow velocity is absent before the pulse arrival, i.e. $v(+\infty) \equiv 0$:

$$v_n \approx -\int_\eta^{+\infty} \frac{1}{c_A\rho}\frac{\partial}{\partial n}(\frac{B_\phi^2}{8\pi})d\eta'. \tag{33}$$

Substituting this expression into the formula for the electric field longitudinal component (27), we finally obtain the following approximate relation:

$$E_\parallel \approx \frac{1}{c}B_\phi\int_\eta^{\infty} \frac{1}{c_A\rho}\frac{\partial}{\partial n}(\frac{B_\phi^2}{8\pi})d\eta'. \tag{34}$$

For order of magnitude estimation, let $\dfrac{\partial}{\partial n} \approx 1/\delta$, where δ is the characteristic transverse scale of the wave, and consider that for a localised pulse with characteristic length λ, the main part of the integral (33) is yielded by the interval $(c_A t - \lambda/2, c_A t + \lambda/2)$. As a result, we can write approximately:

$$E_\parallel \approx \frac{1}{c}\frac{\lambda}{\delta}\frac{B_\phi^3}{8\pi c_A\rho}. \tag{35}$$

Introducing the parameter $\alpha = (\dfrac{B_\phi}{B_z})^3$, we rewrite this expression in the following form:

$$|E_\parallel| \approx |\frac{\lambda}{\delta}\frac{c_A}{c}\frac{\alpha}{2}B_z|. \tag{36}$$

Again using the solution for the case of an exponentially expanding magnetic tube (25) and considering the dependence of the magnetic field components B_ϕ, B_z and the characteristic transverse momentum scale δ on the longitudinal coordinate Z, and assuming that the electric current distribution is homogeneous along the cross section of the tube, we obtain:

$$|\overline{E}_\parallel| \approx |\frac{\lambda}{\delta_0}\frac{c_A}{c}\frac{\overline{\alpha}_0 B_{z0}}{10}|. \tag{37}$$

Here, the bar means the averaging over the magnetic tube cross section and the lower index 0 means the values of the corresponding quantities at the lowest point of the loop, i.e. at $z = 0$. As can be seen from the above formula (37), the absolute value of the non-linear component of the longitudinal electric field remains unchanged in order of magnitude despite the exponential decrease in the amplitude of the Alfvén wave B_ϕ.

Dividing the obtained expression by the value of the Dreicer electric field [10] gives:

$$\frac{|\overline{E}_\parallel|}{E_D} \approx 10^7 \frac{\lambda_0}{\delta_0} \frac{c_{A0} T \overline{\alpha}_0 B_{z0}}{6 n_0} \exp(\frac{z}{H}). \tag{38}$$

At the values of electric currents about $I \approx 10^{10} A$, magnetic field $B_{z0} \approx 10^2 - 10^3$ Gs, electron concentration $n_0 \approx 10^{11} - 10^{13} cm^{-3}$ and plasma temperature $T \approx 10^4 K$ characteristic for the chromospheric parts of the loop, the obtained ratio can already exceed 1 at $z = 0$, which is already necessary for the occurrence of escape electrons. However, the loop expansion leads to an additional exponential factor $\sim \exp(\frac{z}{H})$, which is absent for the magnetic tube of cylindrical geometry with constant cross section considered in [10]. The mentioned factor at characteristic heights of the chromospheric layer ~ 2000 km [20] can increase this ratio by a factor of $10^2 - 10^3$. In particular, if the condition $\frac{\overline{E}_\parallel(h)}{E_D} \geqslant 1$ is fulfilled in the lower part of the chromospheric footpoints, it is moreover fulfilled in the higher region.

The authors of the paper [9] estimated the energy that can be acquired by particles accelerated at a distance of about 100 km in a non-linear electric field with a strength slightly higher than the Dreicer field (0.1 V/cm for the tube configuration considered in the paper, at a plasma concentration $n \approx 10^{11} cm^{-3}$ and plasma temperature $T \approx 10^4 K$). The value obtained was found to be of the order of 1 MeV. However, as can be seen from the results obtained above, the energy of the accelerated particles can be much higher, reaching values of the order of 1 GeV, since in the upper regions of the chromosphere the electric field can significantly exceed the Dreicer limit. The continuum gamma-ray emission and radiation from the decay of neutral pions recorded during the most powerful solar flares indicate that particles with such energies are produced [1, 3] and can be the result of the direct acceleration by the electric field.

5 Conclusions

Thus, the paper presents an analytical study of the propagation of the Alfvén pulse in an axially symmetric magnetic tube of variable diameter. In the linear approximation for a wide range of parameters, it is found that the problem is reduced to the oscillations of a one-dimensional string and has a self-similar solution. Based on the solution found by the perturbation theory, it is shown that an Alfvén pulse of sufficiently large amplitude has a non-linear electric field component directed along the magnetic field of the tube, i.e. along the shape of the tube, leading to the injection of chromospheric plasma and/or high energy particles into the coronal regions of the loop.

The obtained result not only confirms the conclusions gained in [9, 10], but also shows that in an exponentially expanding magnetic tube the excess of this electric field over the Dreicer field, $\frac{\overline{E}_\parallel(h)}{E_D}$, increases in the direction of decreasing plasma concentration (i.e, from the photosphere to the corona) and can reach values of the order of $10^2 - 10^3$. Consequently, the chromospheric footpoints of magnetic loops are favourable to meet the conditions for the generation of the super-Dreicer electric field by Alfvén pulses, which can accelerate electrons to energies of the order of 1 GeV and ensure that the coronal loops are filled with a large number of such high-energy electrons.

A quantitative solution to the problem of the injection of energetic particles into coronal loops requires a detailed analysis of the characteristic parameters and generation efficiency of Alfvén pulses in the lower chromosphere in quiescent, active and preflare regions. Analytical and numerical calculations of the kinetics of the acceleration and escape of high-energy electrons in the electric fields found for an Alfvén pulse of a given amplitude and duration are also required. Finally, it is important to study the non-linear damping of the Alfvén pulse due to its depletion under the backward influence of runaway electrons. In addition, it is of particular interest to study the reflection and passage of Alfvén pulses in the region of the transition layer between the chromosphere and the corona. Among other things, we want to understand to what extent the processes of electron acceleration by the super-Dreicer electric field, and the consequent generation of strong small-scale kinetic turbulence, contribute to the large increase in the effective temperature of the plasma and the heating of the corona as a whole.

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A Appendix

A.1 A.1

Consider the magnetic induction equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) \tag{39}$$

Projecting it on the ϕ-axis, we obtain

$$\frac{\partial B_\phi}{\partial t} = \frac{\partial (\vec{v} \times \vec{B})_r}{\partial z} - \frac{\partial (\vec{v} \times \vec{B})_z}{\partial r} \tag{40}$$

Writing the equation in coordinates, and considering that $v_r \equiv 0$, $v_z \equiv 0$, we get:

$$\frac{\partial B_\phi}{\partial t} = \frac{\partial (v_\phi B_z)}{\partial z} + \frac{\partial (v_\phi B_r)}{\partial r} = B_z \frac{\partial v_\phi}{\partial z} + B_r \frac{\partial v_\phi}{\partial r} + v_\phi (\frac{\partial B_z}{\partial z} + \frac{\partial B_r}{\partial r}) \tag{41}$$

We also have the condition

$$\nabla * \vec{B} = \frac{1}{r} \frac{\partial r B_r}{\partial r} + \frac{\partial B_z}{\partial z} = 0 \tag{42}$$

We finally obtain the first equation in the form:

$$\frac{\partial B_\phi}{\partial t} = B_z \frac{\partial v_\phi}{\partial z} + B_r \frac{\partial v_\phi}{\partial r} - v_\phi \frac{B_r}{r} \tag{43}$$

Now consider the Euler equation for the plasma velocity:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \nabla) \vec{v} = -\frac{1}{\rho} \nabla P - \frac{1}{4\pi\rho} (\vec{B} \times (\nabla \times \vec{B})) \tag{44}$$

Carrying out the projection and also taking into account the conditions $v_r \equiv 0$, $v_z \equiv 0$, we obtain

$$\frac{\partial v_\phi}{\partial t} = -\frac{1}{4\pi\rho} (\vec{B} \times (\nabla \times \vec{B}))_\phi \tag{45}$$

Or

$$\frac{\partial v_\phi}{\partial t} = \frac{1}{4\pi\rho} (B_r (\nabla \times \vec{B})_z - B_z (\nabla \times \vec{B})_r) \tag{46}$$

Finally we have the second equation in the form:

$$\frac{\partial v_\phi}{\partial t} = \frac{1}{4\pi\rho} (B_z \frac{\partial B_\phi}{\partial z} + B_r \frac{\partial B_\phi}{\partial r} + B_\phi \frac{B_r}{r}) \tag{47}$$

A.2 A.2

Let us introduce new coordinates related to the old ones as follows
$\xi(r, z) = r A_\phi(r, z)$, $\eta(r, z) = z$.

Then:

$$\frac{\partial \xi}{\partial z} = -r B_r \tag{48a}$$

$$\frac{\partial \xi}{\partial r} = r B_z \tag{48b}$$

$$\frac{\partial \eta}{\partial z} = 1 \tag{48c}$$

$$\frac{\partial \eta}{\partial r} = 0 \tag{48d}$$

(48e)

Let us express the derivatives on the old coordinates through the derivatives on the new coordinates.

$$\frac{\partial B_\phi}{\partial z} = -rB_r\frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \eta} \tag{49a}$$

$$\frac{\partial^2 B_\phi}{\partial z^2} = r^2B_r^2\frac{\partial^2 u}{\partial \xi^2} - 2rB_r\frac{\partial^2 u}{\partial \xi\partial \eta} + \frac{\partial^2 u}{\partial \eta^2} - \frac{\partial rB_r}{\partial z}\frac{\partial u}{\partial \xi} \tag{49b}$$

$$\frac{\partial B_\phi}{\partial r} = rB_z\frac{\partial u}{\partial \xi} \tag{49c}$$

$$\frac{\partial^2 B_\phi}{\partial r^2} = r^2B_z^2\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial rB_z}{\partial r}\frac{\partial u}{\partial \xi} \tag{49d}$$

$$\frac{\partial^2 B_\phi}{\partial z\partial r} = -r^2B_zB_r\frac{\partial^2 u}{\partial \xi^2} + rB_z\frac{\partial^2 u}{\partial \xi\partial \eta} + \frac{\partial rB_z}{\partial z}\frac{\partial u}{\partial \xi} \tag{49e}$$

$$\tag{49f}$$

Substituting the obtained expressions into the initial equation, we have:

$$\begin{aligned}\frac{\partial^2 B_\phi}{\partial t^2} &= r^2\frac{B_r^2B_z^2}{4\pi\rho}\frac{\partial^2 u}{\partial \xi^2} - 2r\frac{B_z^2B_r}{4\pi\rho}\frac{\partial^2 u}{\partial \xi\partial \eta} + \frac{B_z^2}{4\pi\rho}\frac{\partial^2 u}{\partial \eta^2} - r\frac{B_z^2}{4\pi\rho}\frac{\partial B_r}{\partial z}\frac{\partial u}{\partial \xi} - 2r^2\frac{B_r^2B_z^2}{4\pi\rho}\frac{\partial^2 u}{\partial \xi^2}\\ +2r\frac{B_z^2B_r}{4\pi\rho}\frac{\partial^2 u}{\partial \xi\partial \eta} &+ 2r\frac{B_zB_r}{4\pi\rho}\frac{\partial B_z}{\partial z}\frac{\partial u}{\partial \xi} + r^2\frac{B_r^2B_z^2}{4\pi\rho}\frac{\partial^2 u}{\partial \xi^2} + \frac{B_r^2}{4\pi\rho}\frac{\partial rB_z}{\partial r}\frac{\partial u}{\partial \xi} - r\frac{B_zB_r}{4\pi\rho}\frac{\partial}{\partial z}(\frac{B_z}{\rho})\frac{\partial u}{\partial \xi}\\ &+\frac{B_z}{4\pi}\frac{\partial}{\partial z}(\frac{B_z}{\rho})\frac{\partial u}{\partial \eta} - r\frac{B_r^2}{4\pi}\frac{\partial}{\partial r}(\frac{B_z}{\rho})\frac{\partial u}{\partial \xi} + \frac{B_r}{4\pi}\frac{\partial}{\partial r}(\frac{B_z}{\rho})\frac{\partial u}{\partial \eta} + r\frac{B_z^2}{4\pi}\frac{\partial}{\partial z}(\frac{B_r}{\rho})\frac{\partial u}{\partial \xi}\\ &+r\frac{B_zB_r}{4\pi}\frac{\partial}{\partial r}(\frac{B_r}{\rho})\frac{\partial u}{\partial \xi} + \frac{B_z}{4\pi}\frac{\partial}{\partial z}(\frac{B_r}{r\rho})u + \frac{B_r}{4\pi}\frac{\partial}{\partial r}(\frac{B_r}{r\rho})u - \frac{B_r^2}{4\pi\rho}\frac{u}{r^2}\end{aligned} \tag{50}$$

After adding similar summands we obtain the above expression:

$$\frac{\partial^2 u}{\partial t^2} = \frac{B_z^2}{4\pi\rho}\frac{\partial^2 u}{\partial \eta^2} + \frac{B_z}{4\pi}\frac{\partial}{\partial \eta}(\frac{B_z}{\rho})\frac{\partial u}{\partial \eta} + [\frac{B_z}{4\pi}\frac{\partial}{\partial \eta}(\frac{B_r}{r\rho}) - \frac{1}{r^2}\frac{B_r^2}{4\pi\rho}]u \tag{51}$$

A.3 A.3

Let us differentiate the above expression

$$\frac{\partial u}{\partial \eta} = \frac{\partial f}{\partial \eta}\exp(\frac{1}{2}\int^\eta \frac{1}{B_z}\frac{\partial B_z}{\partial \eta}d\eta) + \frac{1}{2}\frac{1}{B_z}\frac{\partial B_z}{\partial \eta}f\exp(\frac{1}{2}\int^\eta \frac{1}{B_z}\frac{\partial B_z}{\partial \eta}d\eta) \tag{52}$$

Having differentiated the second time, we obtain:

$$\begin{aligned}\frac{\partial^2 u}{\partial \eta^2} &= \frac{\partial^2 f}{\partial \eta^2}\exp(\frac{1}{2}\int^\eta \frac{1}{B_z}\frac{\partial B_z}{\partial \eta}d\eta) + \frac{1}{B_z}\frac{\partial B_z}{\partial \eta}\frac{\partial f}{\partial \eta}\exp(\frac{1}{2}\int^\eta \frac{1}{B_z}\frac{\partial B_z}{\partial \eta}d\eta)\\ +(\frac{1}{2B_z}\frac{\partial B_z}{\partial \eta})^2f\exp(\frac{1}{2}\int^\eta \frac{1}{B_z}\frac{\partial B_z}{\partial \eta}d\eta) &+ \frac{\partial}{\partial \eta}(\frac{1}{2}\frac{1}{B_z}\frac{\partial B_z}{\partial \eta})f\exp(\frac{1}{2}\int^\eta \frac{1}{B_z}\frac{\partial B_z}{\partial \eta}d\eta)\end{aligned} \tag{53}$$

Substituting the obtained expressions into the wave equation and, grouping and bringing similar summands, we come to the expression:

$$\frac{\partial^2 f}{\partial t^2} = \frac{B_z^2}{4\pi\rho}\frac{\partial^2 f}{\partial \eta^2} + \frac{\partial}{\partial \eta}(\frac{B_z^2}{4\pi\rho})\frac{\partial f}{\partial \eta} + [\frac{\partial}{\partial \eta}(\frac{B_z}{4\pi\rho}(\frac{1}{2}\frac{\partial B_z}{\partial \eta} + \frac{B_r}{r})) - \frac{1}{4\pi\rho}(\frac{1}{2}\frac{\partial B_z}{\partial \eta} + \frac{B_r}{r})^2]f \tag{54}$$

By entering the designation:

$$\frac{1}{L} = \frac{1}{B_z}(\frac{1}{2}\frac{\partial B_z}{\partial \eta} + \frac{B_r}{r}) \tag{55}$$

we obtain the above expression. Let us transform the parameter $\frac{1}{L}$ to a more illustrative form. For a fixed value of ξ we have:

$$\frac{1}{L} = \frac{1}{2}\frac{1}{B_z}\frac{\partial B_z}{\partial \eta} + \frac{1}{B_z}\frac{B_r}{r} = \frac{1}{2}(\frac{1}{B_z}\frac{\partial B_z}{\partial \eta} + \frac{2}{r}\frac{\partial r}{\partial \eta}) = \frac{1}{2r^2B_z}\frac{\partial r^2B_z}{\partial \eta} = \frac{\Phi}{2\pi r^2B_z}\frac{\partial}{\partial \eta}(\frac{\pi r^2B_z}{\Phi}) \tag{56}$$

Entering the parameter $\varkappa$ we get the corresponding expression.

A.4 A.4

We obtain the relation between the initial distribution of the azimuthal component of the magnetic field $B_{\phi 0}(r,z)$ and the initial perturbation for vibrations of a one-dimensional string $\psi(\xi,\eta)$.

At the initial moment of time:

$$B_\phi(0,r,z) = B_{\phi 0}(r,z) = B_{\phi 0}(r(\xi,\eta), z(\xi,\eta)) = b(\xi,\eta) \tag{57}$$

On the other hand

$$B_\phi(0,r,z) = u(0,\xi,\eta) = f(0,\xi,\eta)\exp(\frac{1}{2}\int^{\eta}\frac{1}{B_z}\frac{\partial B_z}{\partial \eta'}d\eta') \tag{58}$$

Then:

$$\psi(\xi,\eta) = f(0,\xi,\eta) = b(\xi,\eta)\exp(-\frac{1}{2}\int^{\eta}\frac{1}{B_z}\frac{\partial B_z}{\partial \eta'}d\eta') \tag{59}$$

According to the solution of the wave equation by d'Alambert's method:

$$f(t,\xi,\eta) = \frac{\psi(\xi,\eta-c_A(\eta)t)+\psi(\xi,\eta+c_A(\eta)t)}{2} = \sum_{\pm}\frac{1}{2}b(\xi,\eta\pm c_At)\exp(-\frac{1}{2}\int^{\eta\pm c_At}\frac{1}{B_z}\frac{\partial B_z}{\partial \eta'}d\eta') \tag{60}$$

In this case the solution for the Alfvén wave in the new coordinates is written in the form:

$$u(t,\xi,\eta) = f(t,\xi,\eta)\exp(\frac{1}{2}\int^{\eta}\frac{1}{B_z}\frac{\partial B_z}{\partial \eta'}d\eta') = \sum_{\pm}\frac{1}{2}b(\xi,\eta\pm c_At)\exp(\frac{1}{2}\int_{\eta\pm c_At}^{\eta}\frac{1}{B_z}\frac{\partial B_z}{\partial \eta'}d\eta') \tag{61}$$

Returning to the old coordinates we obtain:

$$B_\phi(t,r,z) = u(t,\xi(r,z),\eta(r,z)) = \sum_{\pm}\frac{1}{2}b(\xi(r,z),\eta(r,z)\pm c_At)\exp(\frac{1}{2}\int_{\eta(r,z)\pm c_At}^{\eta(r,z)}\frac{1}{B_z}\frac{\partial B_z}{\partial \eta'}d\eta') \tag{62}$$

Or

$$B_\phi(t,r,z) = \sum_{\pm}\frac{1}{2}B_{\phi 0}(r[\xi(r,z),\eta(r,z)\pm c_At], z[\xi(r,z),\eta(r,z)\pm c_At])\exp(\frac{1}{2}\int_{\eta(r,z)\pm c_At}^{\eta(r,z)}\frac{1}{B_z}\frac{\partial B_z}{\partial \eta'}d\eta') \tag{63}$$

In the case when $B_z(z)$ does not depend on the radius r, we have:

$$\frac{\partial \xi}{\partial z} = r\frac{\partial A_\phi}{\partial z} = -rB_r = \frac{r^2}{2}\frac{\partial B_z}{\partial z} \tag{64}$$

Choosing a natural calibration for the vector potential $A_\phi(r,z) = \frac{1}{2}rB_z(z)$, we obtain:

$$\xi(r,z) = \frac{r^2}{2}B_z(z) \tag{65}$$

Then, using the obtained relation of new and old coordinates, we finally have the following solution for the Alfvén wave in old coordinates expressed through the initial function $B_{\phi 0}$:

$$B_\phi(t,r,z) = \sum_{\pm}\frac{1}{2}B_{\phi 0}(r\sqrt{\frac{B_z(z)}{B_z(z\pm c_At)}}, z\pm c_At)\exp\,(\frac{1}{2}\int_{z\pm c_At}^{z}\frac{1}{B_z}\frac{\partial B_z}{\partial z'}dz') \tag{66}$$

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